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HIGLU: A Program for the Calculation of the Total Higgs Production Cross Section at Hadron Colliders via Gluon Fusion including QCD Corrections

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Abstract

A program for the calculation of the total Higgs production cross section via gluon fusion at hadron colliders including next–to–leading order QCD corrections is presented. It is suitable especially for Standard Model Higgs bosons and the neutral Higgs particles of the minimal supersymmetric extension. The program provides a model–independent calculation for scalar [$\mathcal{CP}$–even] as well as pseudoscalar [$\mathcal{CP}$–odd] Higgs bosons including the contributions of virtual top and bottom quarks inside the loop coupled to the Higgs particles. The relevant input parameters can be chosen from an input file. As a special case the minimal supersymmetric extension of the Standard Model can be investigated. The corresponding couplings are implemented including the leading higher order corrections.

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1 Introduction

At the future hadron collider LHC the Higgs boson will be produced primarily via the gluon fusion mechanism for the entire relevant Higgs mass range within the Standard Model [$\mathcal{SM}$] [1] as well as the minimal supersymmetric extension [$\mathcal{MSSM}$] [2]. Recently the next–to–leading order QCD corrections to the production cross sections of scalar [$\mathcal{CP}$–even] as well as pseudoscalar [$\mathcal{CP}$–odd] Higgs bosons have been calculated [3, 4]. They are significant for the theoretical prediction of the cross sections leading to an increase by up to a factor of two compared to the lowest order results.

In this paper the program HIGLU[3] for the calculation of the total Higgs production cross sections including these next–to–leading order QCD corrections will be presented. Various relevant input parameters can be chosen from an input file including a flag specifying the model. Possible options are the Standard Model, its minimal supersymmetric extension and a general Higgs model by initializing the Higgs Yukawa couplings to the heavy quarks appropriately. The program includes the contribution of the top and bottom quarks in the loop that generates the Higgs couplings to gluons. Within the Standard Model as well as in most of the parameter space of the $\mathcal{MSSM}$ these contributions provide an excellent approximation for all cases in practice. Moreover, the program allows to calculate the decay widths of Higgs bosons into gluons including next–to–leading order QCD corrections. The gluonic decay mode plays a significant rôle in the intermediate mass range at future e^+e^- colliders [3].

The source code of the program is written in FORTRAN. It has been tested on a VAX–station using the operating system VMS, on different workstations running under UNIX and IBM computers with the operating systems CMS and TSO. The numerical integration is performed by using the VEGAS–package [5] for integrals of dimension up to three. Parton distributions can be attached to the program in any desirable way by adjusting the corresponding subroutine as described in section 5. As the standard parametrization the program contains the GRV sets [6].

2 Results

2.1 $pp \to \Phi + X$

The hadron cross section of Higgs boson[4] production via gluon fusion $gg \to \Phi$ ($\Phi = \mathcal{H}, A$) including next–to–leading order QCD corrections, can be cast into the form

$$\sigma(pp \to \Phi + X) = \sigma^\Phi_{LO} + \Delta\sigma^\Phi_{virt} + \Delta\sigma^\Phi_{gg} + \Delta\sigma^\Phi_{gq} + \Delta\sigma^\Phi_{q\bar{q}} \tag{1}$$

with the lowest order cross sections

$$\sigma(pp \to \Phi + X) = \sigma^\Phi_0 \tau_\Phi \frac{d\mathcal{L}^{gg}}{d\tau_\Phi} \tag{2}$$

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[4]The scalar [$\mathcal{CP}$–even] Higgs particles will generically be denoted by $\mathcal{H}$, the pseudoscalar [$\mathcal{CP}$–odd] by A and all the neutral Higgs bosons by Φ.

The coefficients are

$$\sigma_0^{\mathcal{H}} = \frac{G_F \alpha_s^2(\mu^2)}{288\sqrt{2}\pi} \left| \sum_Q g_Q^{\mathcal{H}} A_Q^{\mathcal{H}}(\tau_Q) \right|^2 \qquad \sigma_0^{A} = \frac{G_F \alpha_s^2(\mu^2)}{128\sqrt{2}\pi} \left| \sum_Q g_Q^{A} A_Q^{A}(\tau_Q) \right|^2 \quad (3)$$

They include the Yukawa couplings g_Q^Φ normalized to the $\mathcal{SM}$ couplings, and the quark amplitudes

$$A_Q^{\mathcal{H}}(\tau_Q) = \frac{3}{2}\tau_Q \left[1 + (1-\tau_Q) f(\tau_Q)\right] \qquad A_Q^{A}(\tau_Q) = \tau_Q f(\tau_Q) \quad (4)$$

The function $f(\tau)$ is defined as

$$f(\tau) = \begin{cases} \arcsin^2 \dfrac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\dfrac{1}{4}\left[\log \dfrac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1 \end{cases} \quad (5)$$

and the scaling variables are

$$\tau_Q = \frac{4m_Q^2}{m_\Phi^2} \qquad \text{and} \qquad \tau_\Phi = \frac{m_\Phi^2}{s} \quad (6)$$

The parameter m_Q denotes the heavy quark mass, m_Φ the Higgs boson mass, s the total center of mass energy squared, G_F the Fermi constant and α_s the QCD coupling constant. The term $\Delta\sigma_{virt}^\Phi$ parametrizes the infrared regularized virtual two–loop corrections and the terms $\Delta\sigma_{ij}^\Phi$ $(i,j = g,q,\bar{q})$ the individual collinear regularized real one–loop corrections corresponding to the subprocesses

$$gg \to \Phi g, \qquad gq \to \Phi q, \qquad q\bar{q} \to \Phi g$$

The expressions for $\Delta\sigma_{virt}^\Phi$ and $\Delta\sigma_{ij}^\Phi$ can be found in Refs.[3, 4]. The gluon luminosity is defined by

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x,Q^2) g(\tau/x, Q^2) \quad (7)$$

where $g(x,Q^2)$ denotes the gluon density. The natural values to be chosen for the renormalization scale μ of the strong coupling $\alpha_s(\mu^2)$ and the factorization scale Q of the parton densities is given by the Higgs mass m_Φ.

The program HIGLU calculates the five terms in eq.(1) contributing to the total cross section separately as well as their sum for all kinds of neutral Higgs bosons Φ. The computation of differential cross sections can be found in Refs.[7] and is not implemented in the program.

3 $\Phi \to gg$

The decay widths of Higgs bosons Φ into gluons up to next–to–leading order are given by

$$\begin{aligned}
\Gamma(\Phi \to gg(g), gq\bar{q}) &= \Gamma_{LO}(\Phi \to gg)\left[1 + E_\Phi \frac{\alpha_s}{\pi}\right] \\
E_\Phi &= E^\Phi_{virt} + E^\Phi_{ggg} + N_F E^\Phi_{gq\bar{q}}
\end{aligned} \tag{8}$$

with the leading–order expressions

$$\begin{aligned}
\Gamma_{LO}(\mathcal{H} \to gg) &= \frac{G_F \alpha_s^2(\mu^2)}{36\sqrt{2}\pi^3}\left|\sum_Q g_Q^{\mathcal{H}} A_Q^{\mathcal{H}}(\tau_Q)\right|^2 \\
\Gamma_{LO}(A \to gg) &= \frac{G_F \alpha_s^2(\mu^2)}{16\sqrt{2}\pi^3}\left|\sum_Q g_Q^{A} A_Q^{A}(\tau_Q)\right|^2
\end{aligned} \tag{9}$$

The amplitudes $A_Q^\Phi(\tau_Q)$ are defined in eq.(4). The coefficient E^Φ_{virt} denotes the infrared regularized virtual two–loop corrections, E^Φ_{ggg} and $E^\Phi_{gq\bar{q}}$ the collinear regularized real one–loop corrections. The analytical formulae of these contributions can be found in Ref.[3]. The parameter N_F fixes the number of light external flavors produced in the decay $\Phi \to gq\bar{q}$, which is defined to be equal to the number of flavors contributing to the QCD β function. This definition maps large logarithms into the running strong coupling $\alpha_s(\mu^2)$. The natural renormalization scale μ of the strong coupling $\alpha_s(\mu^2)$ is given by the corresponding Higgs boson mass m_Φ.

4 Input Parameters

In addition to the source code of the program HIGLU an input file defined as unit 98 is needed, from which the program reads the input parameters. The name of this input file can be defined in the first OPEN statement of HIGLU. It should be noted that the input numbers must *not* start before the equality signs in each corresponding line. The input file contains the following parameters:

process: integer
  choose the process to be calculated:
  0: gluon fusion $gg \to \Phi$
  1: gluonic decay $\Phi \to gg$

collider: integer
  choose the hadron collider mode for the gluon fusion process:
  0: pp
  1: $p\bar{p}$
  This flag is only relevant for the gluon fusion process.

energy: double precision

center of mass energy [in TeV] of the hadron collider for the gluon fusion process

model: integer

choose the model, in which the process should be calculated:

0: $\mathcal{SM}$

1: $\mathcal{MSSM}$ including the dominant two–loop corrections to the Higgs masses and couplings

2: $\mathcal{MSSM}$ including only the the leading one–loop corrections, which increase as the fourth power of the top mass.

3: any model

All stop mixing parameters of the $\mathcal{MSSM}$ are set equal to zero.

tanbeta: double precision

the $\mathcal{MSSM}$ parameter tgβ, which is irrelevant if the flag model equals zero or three.

$g_{b,t}$: double precision

the Yukawa couplings of the corresponding Higgs boson to the top and bottom quarks normalized to the $\mathcal{SM}$ couplings. These couplings are only relevant, if the flag for the model is chosen to be three. If the calculation is performed within the $\mathcal{MSSM}$, these parameters are irrelevant, because the Yukawa couplings are calculated from tgβ and the chosen Higgs mass m_{Higgs}. In the $\mathcal{SM}$ [flag model = 0] these couplings are automatically set equal to unity.

$m_{b,t}$: double precision

pole masses of the bottom and top quarks [in GeV]. Useful values for these masses are given in the sample of the input file in the appendix: $m_t = 176$ GeV, $m_b = 5$ GeV.

type: integer

choose the neutral Higgs boson type:

1: heavy scalar H

2: pseudoscalar A

3: light scalar h

In the $\mathcal{SM}$ this flag is set equal to unity automatically. If the model is chosen to be the $\mathcal{MSSM}$, all three types of Higgs bosons can be chosen. If the flag for the model is set equal to three, only the values 1 and 2 are possible, because the nature of the Higgs boson will be characterized by its mass and its Yukawa couplings $g_{b,t}$ to the heavy quarks.

m_{Higgs}: double precision

the mass [in GeV] of the chosen Higgs boson type.

loop: integer

choose one–loop or two–loop formula for the strong coupling α_s in the $\overline{MS}$ scheme:

1: one–loop
2: two–loop

choice: integer
choose the input value of the strong coupling α_s:
1: α_s is defined by the value for $\alpha_s(M_Z)$
2: α_s is fixed by the QCD scale $\Lambda_{\overline{MS}}^{(N_F)}$

alpha$_S$: double precision
strong coupling α_s at scale of the Z boson mass M_Z. This parameter is relevant for the flag choice = 1.

N_F: integer
the number of flavors, for which the QCD scale $\Lambda_{\overline{MS}}^{(N_F)}$ is given. This parameter can be chosen to be 3,4,5 or 6. The QCD scales for different numbers of flavors are computed by using the quark masses $m_{b,t}$ from the input file and $m_c = 1.5$ GeV via the matching conditions of the $\overline{MS}$ scheme at the thresholds.

lambda: double precision
QCD scale $\Lambda_{\overline{MS}}^{(N_F)}$ in GeV.

The parameters N_F and lambda are relevant for the flag choice = 2.

n_{ext}: integer
number of light external flavors to be included in the gluonic decay width of the Higgs boson. If n_{ext} is set equal to 4, the bottom contribution $\Phi \to b\bar{b}g$ is subtracted, and if n_{ext} equals three, the part $\Phi \to c\bar{c}g$ is removed. This parameter can be chosen to be 3,4 or 5. The running strong coupling $\alpha_s(\mu^2)$ uses the *same* number of flavors.

$mu_{1,2}$: double precision
parameters defining the renormalization scale μ [in GeV] of the strong coupling constant α_s in the following way:

$$\mu = \mu_1 \; m_{Higgs} + \mu_2$$

The variable μ_1 is a dimensionless coefficient of the Higgs mass m_{Higgs}, and μ_2 denotes a fixed scale [in GeV].

$Q_{1,2}$: double precision
parameters fixing the factorization scale Q [in GeV] for the gluon fusion process:

$$Q = Q_1 \; m_{Higgs} + Q_2$$

Q_1 denotes a dimensionless coefficient of the Higgs mass m_{Higgs}, whereas Q_2 defines a fixed scale [in GeV].

abserr: double precision
absolute error for the VEGAS integration.

points: integer

number of points for the VEGAS integration. 10000 points yield a sufficient precision, whereas the results using 1000 points with 5 iterations are already reliable at the percent level.

itmax: integer

maximal number of iterations for the VEGAS integration.

print: integer

flag for the print out style of the VEGAS iterations:
0: no print out
1: pretty print out
10: print out as a table

A sample of the input file is given in the appendix.

5 Structure Functions

For the implementation of structure functions the subroutine STRUC has to be changed appropriately. This part of the program reads as follows:

```
      SUBROUTINE STRUC(X,Q,PDF)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION PDF(-6:6)
      COMMON/FACSC/ISCHEME
C...X          - BJORKEN X
C...Q          - MOMENTUM SCALE  (IN GeV)
C...PDF(-6:6)  - MATRIX CONTAINING  X*P(X,Q)
C...    IPDF = -6 ,  -5 ,  -4 ,  -3 ,  -2 ,  -1 ,0 ,1,2,3,4,5,6
C...         T_BAR,B_BAR,C_BAR,S_BAR,U_BAR,D_BAR,GL,D,U,S,C,B,T

C--- CHOOSE PROTON STRUCTURE FUNCTIONS AND THEIR FACTORIZATION SCHEME
C--- ISCHEME:  0            1
C---           MSBAR        DIS
      ISCHEME=0

      Q2=Q**2
      ISET=2
      CALL PDGRV(ISET,X,Q2,PDF)

      RETURN
      END
```

The line calling the subroutine PDGRV has to be changed, if another parametrization should be used. The flag ISCHEME has to be specified according to the factorization scheme, which has been used for the parametrization of the parton densities. Note that the array PDF has to be generated in accordance with the given convention.

6 Output

The output of the program HIGLU is written to a file unit 99, which contains the chosen input parameters as well as all results obtained by VEGAS integrations. The VEGAS iterations are written to the standard output, if the flag print is set equal to 1 or 10. The integrated results contain all individual contributions of the QCD corrections separately and their total sum. A sample of the output is presented in the appendix, which can be obtained with the input file shown before, if the flag model and the Higgs mass m_{Higgs} are specified appropriatly. If the $\mathcal{MSSM}$ is chosen as the Higgs model, the pseudoscalar Higgs mass m_A corresponding to the scalar Higgs masses $m_{\mathcal{H}}$ is also typed out.

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7 Appendix

7.1 Input File

An example of the input file is given by

```
PROCESS:  0 = GG --> H         1 = H --> GG
=======

PROCESS  = 0
---------------------------------------------------------------
COLLIDER:   0 = P P              1 = P PBAR
========

COLLIDER = 0
---------------------------------------------------------------
TOTAL ENERGY: [TEV]
=============

ENERGY   = 14.D0
---------------------------------------------------------------
MODEL: 0 = SM  1 = MSSM (TWO-LOOP)  2 = MSSM (ONE-LOOP) 3 = ANY
======

MODEL    = 0
---------------------------------------------------------------
TAN(BETA): (MSSM)
==========

TANBETA  = 1.5D0
---------------------------------------------------------------
COUPLINGS:  G_B = BOTTOM    G_T = TOP
==========  (MODEL = 0)

G_B      = 1.D0
G_T      = 1.D0
---------------------------------------------------------------
QUARK MASSES: [GEV]
=============

M_B      = 5.D0
M_T      = 176.D0
---------------------------------------------------------------
```

```
HIGGS TYPE AND MASS [GEV]:  1 = HEAVY SCALAR   2 = PSEUDOSCALAR
==========================  3 = LIGHT SCALAR

TYPE     = 1
M_HIGGS  = 200.D0
---------------------------------------------------------------
SCALES: [GEV] MU = MU_1*M_HIGGS + MU_2:   RENORMALIZATION SCALE
=======        Q =  Q_1*M_HIGGS +  Q_2:     FACTORIZATION SCALE

MU_1     = 1.D0
MU_2     = 0.D0
Q_1      = 1.D0
Q_2      = 0.D0
---------------------------------------------------------------
ORDER OF ALPHA_S:  1 = LO     2 = NLO
=================

LOOP     = 2
---------------------------------------------------------------
DEFINITION OF ALPHA_S:  1 = ALPHA_S (M_Z)   2 = BY LAMBDA (N_F)
======================

CHOICE   = 1
---------------------------------------------------------------
ALPHA_S (M_Z):
==============

ALPHA_S  = 0.118D0
---------------------------------------------------------------
LAMBDA_NF: [GEV] (QCD SCALE)
==========

N_F      = 5
LAMBDA   = 0.226D0
---------------------------------------------------------------
NUMBER OF EXTERNAL LIGHT FLAVORS: (FOR H --> GG)
=================================

N_EXT    = 3
---------------------------------------------------------------
VEGAS:       ABSERR = ABSOLUTE ERROR
======       POINTS = NUMBER OF CALLS
             ITMAX  = NUMBER OF ITERATIONS
```

```
            PRINT  = PRINT OPTION FOR INTERMEDIATE VEGAS-OUTPUT
                     0            1              10
                     NO OUPUT     PRETTYPRINT    TABLE

ABSERR   = 0.D0
POINTS   = 10000
ITMAX    = 5
PRINT    = 10
--------------------------------------------------------------
```

7.2 Output

7.2.1 Gluon Fusion

```
 VEGAS:
 ======
 ABSERR    =    0.000000E+00
 POINTS    =      10000          ITERATIONS   =     5

 GLUON FUSION: GG --> HIGGS
 ==========================

 P P COLLIDER
 ============
 ENERGY    =     14.0000     TEV

 LAMBDA_5  =    0.226232     GEV     NLO-ALPHA_S (M_Z) =    0.118000
 REN-SCALE =     200.000     GEV     FAC-SCALE =     200.000     GEV

 T-MASS    =     176.000     GEV     B-MASS    =     5.00000     GEV

 HIGGS     = H
 ============================================================
 M_H       =     200.000      GEV
 G^H_B     =     1.00000      G^H_T    =     1.00000
 SIG_LO    =     6.42484     +-    0.681792E-05  PB
 SIG_VIRT  =     3.45509     +-    0.366648E-05  PB
 SIG_GG    =     4.78077     +-    0.466710E-02  PB
 SIG_GQ    =   -0.268282     +-    0.709725E-03  PB
 SIG_QQ    =    0.182952E-01 +-    0.173479E-04  PB
 SIG_NLO   =     14.4107     +-    0.472080E-02  PB
```

7.2.2 Gluonic Decay

```
VEGAS:
======
ABSERR    =    0.000000E+00
POINTS    =      10000          ITERATIONS   =     5

HIGGS --> GG
============

NF_EXT    =     3

LAMBDA_5  =    0.226232     GEV     NLO-ALPHA_S (M_Z) =    0.118000
REN-SCALE =     500.000     GEV

T-MASS    =     176.000     GEV     B-MASS    =     5.00000     GEV

MSSM (2-LOOP):  TG(BETA) =  1.50
==============
 Z-MASS =   91.187     GEV           W-MASS   =   80.330     GEV
 NO MIXING                        SUSY-SCALE =   1000.0     GEV

HIGGS     = H
=============================================================
M_H       =     500.000      GEV
M_A       =     490.788      GEV
G^H_B     =     1.47436      G^H_T    =   -0.691622
GAM_LO    =     9.15160      MEV
E_VIRT    =     15.2434
E_GGG     =     7.68668     +-    0.812334E-04
E_GQQ     =   -0.972425     +-    0.307239E-04
E_TOT     =     20.0128     +-    0.122860E-03
GAM_NLO   =     13.9932     +-    0.297226E-04  MEV
```

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